National Seminar on Jain Mathematics: Historical and Theoretical Aspects,, JVBI Ladnun, Oct 2009

Jain Mathematics: Historical and Modern Aspects

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Abstract

In ancient India Mathematics was of major interest to intellectuals, particularly philosophers. There were different schools of mathematics such as Hindu, Jain and Buddhist as early as third B.C.E. In the Jaina School, two systems were developed on the basis of mathematics, (1) Karma system, and (2) Cosmological system. Both Svetambara and Digambara texts containing system theoretical approaches of Karma claim to be based on Purvas, the reportedly oldest and most authentic Jaina texts, or on parts thereof, such as the lost twelfth Anga, the Dristivada. The paper reviews the major contributions made by Jain Acaryas in the field of Mathematics.

In ancient times Jaina Acaryas and other scholars employed mathematics to support philosophical concepts. With the advent of modern science both science and mathematics have become important for philosophical thought The mathematical contributions made by Jain scholars are more of historical importance today but the principles and theories for which they were developed are of great importance and are highly relevant even in context with the modern science We illustrate this point with the help of two examples, one of micro cosmology and the other of doctrine of karma, in this paper.

Jain philosophy provides a comprehensive picture of micro cosmology and the modern science has only explored a part of it. When the science discovers the whole range of existence, the world view will undergo a sea change giving rise to new theories and principles and abandonment of some of the existing theories. The intelligence of atman constructs the body according to the blue print contained in the karma body and the cell received from parents.

It is a happy sign that some aspects of doctrine of karma are now being validated in some form by scientific research. Once the doctrine of karma is validated, Jain philosophy shall be the option left to scientific community and then science shall merge with philosophy

giving rise to realization of the true goal of life and spirituality shall prevail over the materialistic approach that is the cause of misery, dissatisfaction and non-peace.

Jain Mathematics: Historical and Modern Aspects

In ancient India Mathematics was of major interest to intellectuals, particularly philosophers. There were different schools of mathematics such as Hindu, Jain and Buddhist as early as third B.C.E. In the Jaina School, two systems were developed on the basis of mathematics, (1) Karma system, and (2) Cosmological system. The earliest Jaina texts on karma, among the Svetambara Agama, generally dated fourth or third century B.C.E., are exclusively concerned with ethics. The later Digambara and Svetambara texts of from the beginning of the first millennium C.E. contain cosmological speculation and technical philosophical and mathematical detail. The first fully developed system of karma is in evidence in the Digambara "Siddhanta" and in elementary form in late canonical Svetambara scriptures, such as Samavaya, Viyahapannati and Arya Syama's Pannavana (c. 79 B.C.E.), whose contents overlap in parts with the much more elaborated Satkhandagama of the Digambara's Puspadanta and Bhutabali (c. 100-200 C.E.), according to Dalsukh D. Malvaniya. The later theoretical works of the Digambaras and the Svetambaras works such as Karmaprakriti (Kammapayadi) of Sivasarmasuri, Pancasamgraha (Pancasmgaha) of Candrasi Mahattara and Karmagranthas of the thirteenth century Devendrasuri, all composed in Prakrit, are largely predicated on these earlier scriptures, but are more systematic with new details added, in particular on the mechanisms underlying gunasthana (gunatthanas) and their mathematical quantification, apparently taken from Purva (Puvva) texts such as Kasayapahuda (Kasayaprabhrita). Both Svetambara and Digambara texts containing system theoretical approaches of Karma claim to be based on Purvas, the reportedly oldest and most authentic Jaina texts, or on parts thereof, such as the lost twelfth Anga, the Dristivada.

The earliest known astronomical works of Jainas are supposed to be Surya- Prajnapti, Candra- Prajnapti and Jambudvipa - Prajnapti, which were composed as early as 5th century

B.C. The Jaina canonical works, Uttaradhyayarna (300 BC) and Bhagavati - Sutra (300 BC) describe mathematical results, relating to varga, ghana, combinations, etc. Bhadrabahu (300 BC) the last Srutakevali, composed two astronomical works, namely, a commentary on the Surya - Prajnapti and Bhadrabahu Samhita. Bhadrabahu - II, supposed to be younger brother of Varahamira, also wrote a book Bhadrabahu Samhita.

Umaswati (or Umaswami) (150 BC) composed Tattvarthadhigama - Sutra- Bhasya which also contains some mathematical results. Anuyogadvara Sutra (1^{st}c B.C.) describes 1st square, 2nd square etc. which shows that Jaina scholars had the idea of indices in the first century B.C.

Yativrishabha compiled both the system theories. The karma system appears first in Kasayapahuda of Gunadharacarya which deals with biological system. The cosmological system is found in Tiloyapannati of Yativrishabha. It is generally believed that he lived in 5^{th}-6^{th} C.A.D. and was contemporary of mathematician Aryabhatta. Some scholars, however, place him in 2nd C.A.D. Tiloyapannati contains geometry, arithmetic, algebra, arithmetic progression, geometric progression, etc.

Sarvanandi (458 AD) wrote Lokabibhaga Grantha, which contains several mathematical theories, numeration, algebra and astronomical calculations.

Haribhadra Suri (750 A.D.) wrote at least two astronomical works, Laghusanghayani (or Jambudvipa Samgrahani) and a commentary on Anuyogadvara of Aryaraksita.

Virasenacarya of the 8th century A.D. wrote a commentary, Dhavala (816 AD) on Satkhandagama of 1st century AD. Dhavala includes different aspects of mathematics such as algebra, series (Parikarma), application of large numbers, infinite numbers, analysis of numbers, etc. With the help of mathematical principles he could explain the Jaina phisolophy in a scientific manner.

Under the patronage of king Amoghavarsa (815-877C.E.) of Rastrakuta dynasty the Jain scholar Mahaviracarya authored the mathematical work Ganitasara Samgraha in Sanskrit (about 850 C.E.). The book is a rich source of information on ancient Indian Mathematics and was used as a text - book for centuries in South India. The book is a collection of arithmetic, algebra, geometry, astronomy, etc.

Silanka (820 AD) wrote commentary on Anuyogadvara Sutra where he quoted some Prakrita gathas relating to Permutation and Combination.

Sridhara (10th century AD) of Karnataka, who became Jain in later life, wrote Ganit - sara and Jyotirjnana Vidhi in Sanskrit.

Acharya Nemichandra Siddnanta Chakravartin (10th century AD) who was Pontiff of Camundaraya, General and Prime Minister of famous Ganga Empire, composed the mathematical work, 'Triloksara' that contains mathematical formulae. He expressed numbers as powers of 2 and thus nearly developed a logarithmic system with the base 2. He also described different kinds of Series (Dhara). His other important works are Grommatasara and Labdhisara. He described the Rule of Three which was later transmitted to Arabs probably in the eighteenth century and from there it went to Europe. Prof. L.C. Jain considers Gommatasara as a synopsis of Satkhandagama, Labdhisara and Ksapansara as summaries of Kasayapahuda and Trilokasara as a summary of Tiloyapannati.

Malayagiri (11th century AD) wrote commentaries on Surya-pannati, Jambudvipa - pannatti, Candra - pannatti, Jyotiskarandaka, Vrihat - Ksetrasamas, Vrihat - Samgrahani, etc. He made use of several mathematical and astronomical formulae and principles.

Rajaditya (1120 - 1190 AD), a disciple of Subhacandradeva, was the head Pandita in the court of King Vishnuvardhana. He was a poet as well as a mathematician and wrote in Kannada, Vyavaharaganita, Vyavahararatna, Ksetra - ganita, Jainaganita sutra-tikodaharana, Citrahasuge, Lilavati, Ganitavilas and Ganita Samgraha. He gives many examples of mathematical tables, trigonometry, arithmetic of compound interest and other subjects.

The poet and scholar Kumudendu (1275A.D.), said to be disciple of Virasena and Jinasena of the famed Dhavala, composed 'Siri Bhuvalaya' that contains a number of mathematical theories and problems. Kumudendu's work seems to be more advanced than even Virasena and is not easy of comprehension.

Thakkura Pheru (1265-1330 AD) wrote a mathematical work named Ganita Sara Kaumudi in Prakrta which is based on two earlier works, Lilavati of Bhaskara-II and Ganit Sara - Samgraha of Mahaviracarya and is supposed to be an important work of that time.

Naracandra Upadhyaya (13th century AD) a disciple of Simha Suri of Kasadruhagaccha, was a prolific writer and wrote a number of mathematical and astronomical works.

Mahendra Suri, a disciple of Madana Suri of Bhrguphara, and Chief Pandita in the Court of Firoze Shah Tugalaka, wrote a work on Yantra - raja in 1270 AD which contains five chapters on different aspects of astronomy.

Simhatilaka Suri, a distinguished astronomer of the 13th century AD, and a disciple of Vivudhcandra Suri, authored at least two astronomical and mathematical texts, namely, Bhuvandipika Tika or Vrtti (1269 AD) on Bhuvandipika of Padmaprabha Suri and Ganitatilaka Vrtti on Sripati's Ganitatilika.

Mahimodaya, a disciple of Labdhivijaya Suri, was a great scholar of mathematics and astronomy and wrote three works, namely, Jyotisararatnakara, Ganit Satha Sau or Ganitsara Sauvstabha (1630 AD) and Pancanga - nayanavidhi.

Hemaraja (17th century AD) composed a mathematical work named Ganit Sara in Hindi.

Pt. Todaramal (1740-1768 AD) of Jaipur wrote several works, original or commentary. These include Artha - Samdrsti Adhikara, Gommatasara- tika, Labdhisara- Ksapanasara tika, Samyag Jhana Candrika - tika, Triloka Sara tika, etc.

In ancient times Jaina Acaryas and other scholars employed mathematics to support philosophical concepts. With the advent of modern science both science and mathematics have become important for philosophical thought. In the present times Acarya Mahapragya, Muni Mahendra Kumar and Acarya Kanaknandhi are spearheading scientific and mathematical writings on Jain philosophy.

Many mathematical concepts found in Jaina texts have not been given due credit for their invention. For example, Ganitsara Samgraha of Mahavira (850 CE) describes Fibonacci Numbers and Fibonacci Sequences and general formula for combination, Tiloyapannati and Dhavala describe formula for Logarithms, Dhavala describes 'rasi' meaning Sets and Discontinued Fractions, Aptamimansa of Samantbhadra describes the concepts of probability by the term avyaktva, etc. Notions about decimal place- value notation, theory of indices,

logarithms, infinity etc. are found in Dhavala. Permutation and Combinations are described by Acarya Jayadeva (before 900 CE).

The study of relation between the circumference and diameter of a circle has been of great mathematical interest. The constant ratio now known as Pi finds mention in many ancient and medieval Jain works and is generally given a value equal to $\sqrt{10}$.

In Indian system ten has been the basis for counting since very early days. Later on it served the base for the place value system of numerals which was invented in India about two thousand years ago. Specific names are found for quantities equal in value to 10^n, where n=0, 1, 2, .etc. These decuple terms were later used as names for various notational places when positive integral numbers were written in decimal place- value system. A list of 18 decuple terms was generally accepted by many leading Indian mathematical authors but Mahaviracarya is said to have described a list of 24 decuple terms. Another Jain author, Rajaditya (1190 C.E.) extended the list of decimal place- names to 40^{th} order.

It is seen from above that Jainacaryas attached great importance to the culture of astronomy and mathematics. The mathematical contributions mainly relate to Parikarma (four fundamental operations of arithmetic), Vyavahara (determination), Rajju (rope, meaning geometry), Rasi (Rule of Three), Kalasavarna (operations with fractions), Yavat- tavat (as many as; meaning simple equations and providing with the algebraic symbol, ya), Varga (square, meaning quadratic equations), Ghana (cube, meaning cubic equations), Varga- Varga (bi-quadratic equations) and Vikalpa (Permutation and Combination). The value of pi and knowledge of decimal place- value system of numeration and arithmetic was developed. The conception of infinity, at least five types, was developed to deal with very large measurement of time and space. The powers of numbers and laws of indices were used and a logarithmic system with base 2 was developed. The first recorded instance of the solution of a quadratic equation in India is found in the Tattvarthadhigama Bhasya of Umaswati. Series Mathematics was used in cosmography and AP and GP were employed for accounts of hells and heavens. Mahaviracarya, for the first time, gave a systematic treatment of GP and a method to sum up arithmetic-geometrical series. He also gave general formulae for Permutation and Combination.

A school of Mathematics and Astronomy existed in Kusumpura (modern Patna) from about 3rd century B.C. and it is believed that Jain saint Bhadrabahu was one of its exponents. This school survived till the end of 5th century A.D. when Aryabhata-I wrote his famous text Aryabhatiya. The influence of the school continued for several centuries even after Aryabhata - I. Two other important centers of mathematical culture developed at Ujjain and Mysore. The former had scholars like Brahmagupta and Bhaskara-II and the later had Jain mathematician Mahaviracarya.

Another important contribution of Jainas is in the field of statistics. According to the famous statistician Dr. P.C. Mahalanobis, foundation of the basic principles of statistics has roots in the Jaina theory Syadavad. The Jaina concepts of Syadavad, non-absolutism, seven-predicates, transformative- permanence, etc. have direct relation to the theory of probability, concept of an individual in relation to the population, association, correlation, concomitant variation, stochastic processes, concept of uncertain inference, etc.

The mathematical contributions made by Jain scholars are more of historical importance today but the principles and theories for which they were developed are of permanent importance and are highly relevant even in context with the modern science. The theories and postulates propounded by omniscient are based on reality and are true in all conditions. This is the real strength of Jain philosophy, which has the capacity to serve as a light - house even to the modern science. We shall illustrate this point with the help of two examples, one of micro cosmology and the other of doctrine of karma.

The world we see is nothing but interplay of Jiva and pudgala. Jiva and pudgala have independent existence but help each other and change their modes so as to produce the life activity, without which this world would loose its meaning. Jain philosophy has described both jiva and pudgala in great detail, the pudgala ultimately consists of paramanu whose properties and characteristics are known, and the combination of jiva and pudgala is governed by the doctrine of karma that forms the backbone of Jain philosophy.

The pudgala paramanu has two basic touch properties but when it clubs with another paramanu, the group has four basic touch properties. A paramanu is an energy point and is too small to be of any practical use. In fact, only a group containing infinite paramanus,

known as vargana, has a practical application. Varganas are classified based on the number of paramanus in the group, which is always greater than small infinity. As the number of paramanus in the group increases, there reaches a stage when the total energy of the group condenses to produce mass. The varganas having mass now had eight touches, the additional four touches are produced because of the condensation process. Thus there are two kinds of varganas, one having four touches and no mass, and second having eight touches and mass. The ordinary sun light belongs to the second category, being lowest in that order, and is considered by science to consist of photons, the quantum of energy. It can be seen that in view of Jain philosophy the photon is an aggregate of infinite number of paramanus and is not the smallest unit of energy as assumed by science.

Jain philosophy has classified varganas from two points of views, one the varganas which are useful to jiva and the other a general classification covering the entire range. The first classification describes eight types of vargana and the second gives 23 types. From the point of science the second classification is more useful. According to this we have one class of vargana that is mass less and has four touches; this class includes ahara vargana, tejas vargana, mano vargana and karman vargana, which are used by jiva to construct his subtle body. The varganas having eight touches and mass compose various kinds of mass entities ranging from energy units to sub atomic particles that constitute structures which are being studied by science. Two of these later types of varganas are also supposed to constitute the bodies of one-sense jivas. Many kinds of varganas, particularly those belonging to category one, are still a mystery to science. Thus, Jain philosophy provides a comprehensive picture of micro cosmology and the modern science has only explored a part of it. When the science discovers the whole range of existence, the world view will undergo a sea change giving rise to new theories and principles and abandonment of some of the existing theories.

The doctrine of karma constitutes the laws that jiva follows. Jiva, though an indivisible unit, is supposed to consist of innumerable mathematical parts called predasa, just to explain the operation of the laws. Jain philosophy describes in detail how the karma vargana, existing in whole cosmos, bind with each atman pradesa as karma, remain in existence for a certain length of time, bear fruits on rising and how the karmas migrate with jiva from one body to

another. The karmas bind with each atman pradesa uniformly and also rise uniformly so that all pradesa of atman carry the same amount of karma at any instant.

The karmas are of eight main types; these are grouped in two categories, one, psychical karmas that determine the psychical personality of the jiva, and second the physiological karma that determines the physical personality. The biological science deals only with the physical personality of organisms; the psychical personality of five-sense jivas is studied by the discipline of psychology. Scientific studies show that the physiological karma emit very weak light, called bio photons which are supposed to regulate and control the chemical activity in cells and thereby the entire physiology of the body.

An average grown up human body contains about 60 trillion cells. The DNA in each cell has 30000-40000 genes and one gene has about 3000 bases. The Human Genome thus has about 3 billion bases. This gives about 180×10^{21} bases in the human body. The atman is also supposed to contain innumerable parts mathematically (otherwise it is an indivisible unit). Each part of atman contains total and identical karma. The value of the innumerable is supposed to be more than 10^{140} according to Acharya Kanaknandhi. We see that the number of atman divisions out numbers heavily the number of genes and the number of bases. Let there be 10^{n} atman divisions per base of the gene. As all divisions of atman are identical we may think of one- to - one correspondence between genes and karma divisions. So the karma and genes are locally related, each gene is interacting with karma individually. The bio photons emitted by karma field control and regulate the activity of the gene on an individual basis. The activities in different cells are also related and connected. As a result of coherence of karma field fast communication takes place at all levels within cells and between cells and a complete and perfect coordination is established in the activities of all cells, so that the body behaves as a single unit performing goal- oriented functions. The DNA in every cell is identical but each cell performs differently and produces a variety of proteins in different parts of the body. This kind of selective function of DNA is possible due to karma. There is laboratory evidence that DNA is influenced and reprogrammed by radio and light frequencies. The karma radiations in each gene are identical but they work selectively, they regulate the non-protein making part of genes through a process of selection, and determine different function of cells suitable to their locations in the body. In this manner the performance of the body at the cell level is regulated by atman through karma.

The above proposition is supported by scientific research. Guenter Albrecht-Buehler claims that 30 years of his research on cell has shown that mammalian cells possess intelligence. An intelligent cell contains a compartment, which is capable of collecting and integrating a variety of physically different and unforeseeable signals as the basis of problem solving decisions. According to him, the motile machinery of cells contains sub domains ('microplasts') that can be isolated from the cell and then are capable of autonomous movements. Yet, inside the cell they do not exercise their ability. The situations is comparable to a person's muscles that are capable of contraction outside a person's body, but do not contract at will once they are part of the person, suggesting that they are subject to a control center. The cell as a whole is capable of immensely complex migration patterns for which their genome cannot contain a detailed program as they are responses to unforeseeable encounters.

Cells can 'see', i.e. they can map the directions of near-infrared light sources in their environment and direct their movements toward them. No such 'vision' is possible without a very sophisticated signal processing system ('cell brain') that is linked to the movement control of the cell. There is evidence that cells use 'centrioles' to see all objects around them that emit or scatter near infrared light. Cells reach out for each other across a distance of one cell diameter or more. The natural emitters of near-infrared signals are not yet known. The cells coordinate their bodies during movement and respond to physical and topological parameters which are too weak to have caused the observed reaction by force. These responses are then analyzed and used to identify the nature and location of a cellular data-integration system that may be responsible for these locations.

If cells were intelligent, molecules and their genes would be the 'collaborators' or even 'slaves', but not the 'masters' of the life functions of cells. If cells were intelligent we would have to rethink all the cause-and-effect chains from genes to molecules to cell functions that we believe today to be true. If the cells were intelligent, an organism would be ecology of a huge population of intelligent individuals and we would have to look at the structures and functions of our bodies as the result of the interaction of a huge population, 1000 times the population on Earth, of intelligent individuals.

G.de Purucker wrote some seventy years ago about life atoms, centrosomes, and centrioles. He stated that “In each cell there is a central pranic nucleus which is the life-germ of a life-atom, and all the rest of the cell is merely the carpentry of the cell built around it by the forces flowing forth from the heart of this life-atom.” A life-atom is a consciousness-point. H.B.Blavatsky also said in her article in 1890 that every atom is a little universe in itself; and that every organ and cell in the human body is endowed with a brain of its own, with memory, therefore, experience and discriminative powers.

The intelligence, according to Jain philosophy, means presence of atman pradesa in the cell. The physiological karma, particularly the morphological and feeling producing karma, exercise control through radiations on the working of the genes and the cell functions. The control is local through karma of that cell but at the same time it is also global as the same atman and identical karma are present in all cells; there is a central authority that monitors, coordinates and controls the activities of individual cells as well as of a group of cells like tissue or organ or part, so that each cell, group of cells and organs perform according to plan contained in the karma body. It is clear that the intelligence of atman constructs the body according to the blue print contained in the karma body and the cell received from parents.

It is a happy sign that some aspects of doctrine of karma are now being validated in some form by scientific research. Once the doctrine of karma is validated, Jain philosophy shall be the option left to scientific community and then science shall merge with philosophy giving rise to realization of the true goal of life and spirituality shall prevail over the materialistic approach that is the cause of misery, dissatisfaction and non-peace.

I thank the organizers of the Seminar for giving me this opportunity to speak to you in this august gathering and share some of my views on the important subject of Jain Mathematics on which you are going to deliberate in detail.

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